rotated in space.

4.4.2 Time Dilation

Perhaps the most unexpected consequence of the Lorentz transformation is the way in which our 'commonsense' concept of time has to be drastically modified. Consider a clock C' placed at rest in a frame of reference S' at some point x' on the X axis. Suppose once again that this frame is moving with a velocity v_x relative to some other frame of reference S. At a time t'_1 registered by clock C' there will be a clock C_1 in the S frame of reference passing the position of C':

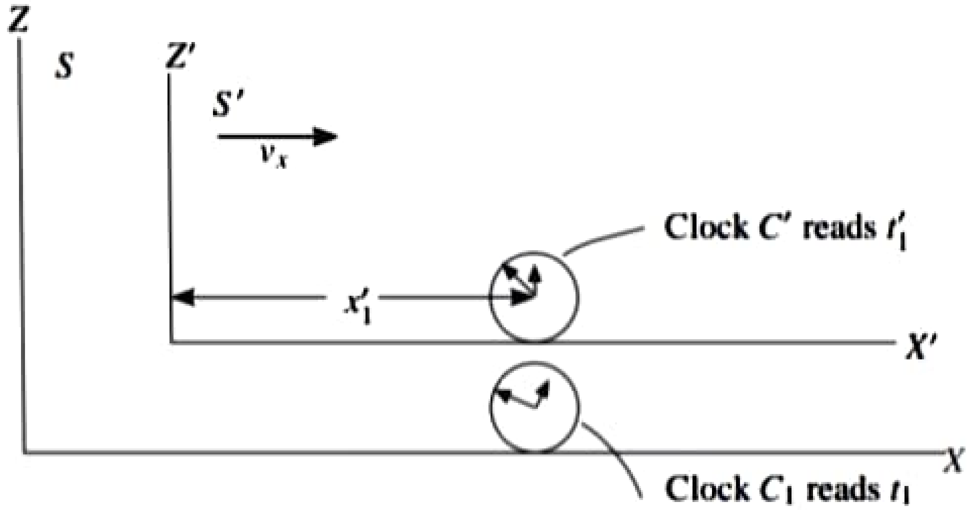
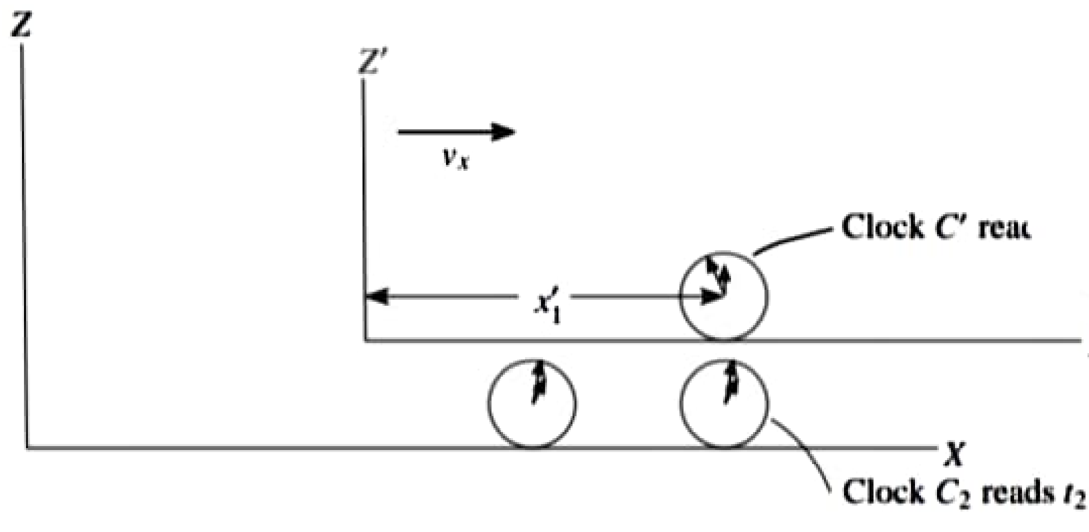


Figure 4.5: Clock C' stationary in S' reads t'_1 when it passes clock C_1 stationary in S, at which instant it reads t_1.

The time registered by C_1 will then be given by the Lorentz Transformation as

$$t_1 = \gamma(t'_1 + v_x x'/c^2). \tag{4.45}$$

Some time later, clock C' will read the time t'_2 at which instant a *different* clock C_2 in S will pass the position x'_1 in S'.



This clock C_2 will read

$$t_2 = \gamma(t_2' + v_x x'/c^2). \tag{4.46}$$

Thus, from Eq. (4.45) and Eq. (4.46) we have

$$\Delta t = t_2 - t_1 = \gamma(t_2' - t_1') = \gamma \Delta t'. \tag{4.47}$$

Once again, since

$$\gamma = \frac{1}{\sqrt{1-(v_x/c)^2}} > 1 \text{ if } v_x < c \tag{4.48}$$

we have

$$\Delta t > \Delta t'. \tag{4.49}$$

In order to interpret this result, suppose that $\Delta t'$ is the time interval between two 'ticks' of the clock C'. Then according to the clocks in S, these two 'ticks' are separated by a time interval Δt which, by Eq. (4.49) is $> \Delta t'$. Thus the time interval between 'ticks' is longer, as measured by the clocks in S, than what it is measured to be in S'. In other words, from the point of view of the frame of reference S, the clock (and all the clocks in S') are running slow. It appears from S that time is passing more slowly in S' than it is in S. This is the phenomenon of *time dilation*. A clock will be observed to run at its fastest when it is stationary in a frame of reference. The clock is then said to be measuring *proper time*.

This phenomenon is just as real as length contraction. One of its best known consequences is that of the increase in the lifetime of a radioactive particle moving at a speed close to that of light. For example, it has been shown that if the lifetime of a species of radioactive particle is measured while stationary in a laboratory to be T', then the lifetime of an identical particle moving relative to the laboratory is found to be given by $T = \gamma T'$, in agreement with Eq. (4.47) above.

Another well known consequence of the time dilation effect is the so-called twin or clock paradox. The essence of the paradox can be seen if we first of all imagine two clocks moving relative to each other which are synchronized when they pass each other. Then, in the frame of reference of one of the clocks, C say, the other clock will be measured as running slow, while in the frame of reference of clock C', the clock C will also be measured to be running slow[5]. This is not a problem until one of the clocks does a U-turn in space (with the help of rocket propulsion, say) and returns to the position of the other clock. What will be found is that the clock that 'came back' will have lost time compared to the other. Why should this be so, as each clock could argue (if clocks could argue) that from its point of view it was the other clock that did the U-turn? The paradox can be resolved in many ways. The essence of the resolution, at least for the version of the clock paradox being considered here, is that there is not complete symmetry between the two clocks. The clock that turns back must have undergone acceleration in order to turn around. The forces associated with this acceleration will only be experienced by this one clock so that even though each clock could argue that it was the other that turned around and came back, it was only one clock that experienced an acceleration. Thus the two clocks have different histories between meetings and it is this asymmetry that leads to the result that the accelerated clock has lost time compared to the other. Of course, we have not shown how the turning around process results in this asymmetry: a detailed analysis is required and will not be considered here.